Final Exam

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Total: 100 points (maximum score including bonus is 105%)

1. Answer the following questions. **(15 points)**

(a) Define *quantificational validity*.
(b) Can a truth-functionally invalid argument be quantificationally valid? Explain.
(c) List the logical operators of PL.
(d) In the truth-tree method, the rule ∃D requires that the instantiating constant **a** be foreign to the branch on which the rule is applied. Explain why this condition is required.
(e) Which of the following are sentences of PL? If they are not, explain why.
 (I) $(\exists x)Fax \vee (\forall a)Faa$
 (II) $\sim (\exists w)(\exists x)(Fwax \equiv Gwaz)$
 (III) $(\forall x)(\forall y)Fxy \supset (\exists w)Gwyx)$

2. Symbolize the following sentences using the symbolization key given. **(10 points)**

$$\begin{aligned} \text{UD} &: \text{Everything} \\ Px &: x \text{ is a person} \\ Tx &: x \text{ is a time} \\ Fxyz &: x \text{ fools } y \text{ at } z \end{aligned}$$

(a) Someone is never fooled.
(b) No one can fool all of the people all of the time.

3. Determine the truth-value of these sentences on this interpretation: **(10 points)**

$$\begin{aligned} \text{UD} &: \text{Set of positive integers} \\ Nx &: x \text{ is greater than 16} \\ Dxyz &: x \text{ plus } y \text{ equals } z \\ Ex &: x \text{ is even} \\ t &: 10 \\ s &: 7 \end{aligned}$$

(a) $(\exists x)(\exists y)((Ex \ \& \ Ey) \ \& \ Dtxy)$
(b) $(\exists x)(Ex \equiv \sim Dtss)$
(c) $(\forall x)Nx \supset Es$
(d) $(\forall x)(Nx \equiv x \neq t)$
(e) $(\forall x)(\forall y)(\exists z)(Dxyz \ \& \ (\forall w)(Dxyw \supset w = z))$

4. Show that the following argument is not quantificationally valid, both by the interpretation method and by the truth-functional method. **(10 points)**

$$\begin{array}{l} (\forall y)(\exists x)(Py \supset Cyx) \\ (\exists x)Cxx \\ \hline (\exists x) \sim Cxx \end{array}$$

5. Consider the following argument: **(15 points)**

$$\begin{array}{l} (\forall x)(Nx \supset (\exists y)Rxy) \\ \sim (\exists x)Rxx \ \& \ Na \\ \hline (\exists y)Ray \end{array}$$

Determine whether it is valid or invalid using each of the following methods:

(a) With the interpretation method;

(b) With a truth-tree;

(c) With a derivation in PD (or $PD+$ if you prefer).

6. Answer the following questions with a *systematic* truth-tree. **(15 points)**

(a) Determine whether $((\exists x)Fax \ \& \sim (\exists x)Fxa) \supset (\forall x)(Gxa \supset x \neq a)$ is quantificationally true, false, or indeterminate.

(b) Determine whether the set $\{(\forall x)(\exists y)((Fxy \ \& \ Rax) \ \& \ Bx), (\exists y)(\forall x)Fxy\}$ is quantificationally consistent.

(c) Determine whether the sentences $(\forall x)(Fx \supset (\forall y)Gy)$ and $(\forall x)(\forall y)(Fx \supset Gy)$ are quantificationally equivalent.

7. Answer the following questions with a natural deduction. **(15 points)**

(a) Show that the following argument is valid in PD (or $PD+$):

$$\begin{array}{l} (\forall x)(Fx \supset (\exists y)Fxy) \\ (\forall x)(\forall y)(Gxy \supset Hxy) \\ \sim (\exists x)(\exists y)Hxy \\ \hline \sim (\exists x)Fx \end{array}$$

(b) Show that $(\forall x)(\forall y)(Fxy \equiv \sim Gyx)$ and $(\forall x)(\forall y) \sim (Fxy \equiv Gyx)$ are equivalent in PD (or $PD+$).

(c) Show that the set $\{(\forall x)(Fx \supset ((\exists y)Gy \supset (\forall y)Gy)), (\exists x)(Fx \ \& \ Gx), (\exists y) \sim Gy\}$ is inconsistent in PD (or $PD+$).

8. Symbolize and determine whether it is valid by the method of your choice. Make sure that you fully specify the symbolization key. **(10 points)**

(a) Some scientific subjects are not interesting, but all scientific subjects are edifying. Therefore, some edifying things are not interesting.

(b) If anyone likes Frege, then she does not like Hegel. Everyone either likes Hegel or likes Russell. Some don't like Russell. But people like Russell, unless they are insane, in which case they like Hegel. Therefore, anybody likes Frege unless she is insane.

Bonus (5 points): Using the concepts of *satisfaction* and *variable assignment*, determine whether the sentences $(\forall x)Fx \vee (\forall x) \sim Fx$ and $(\forall y)(Fy \supset y = b)$. If they are not equivalent, provide an *extensional* interpretation (*i.e.*, an interpretation in which predicates are assigned sets of n-tuples) on which their truth-value differ.

Derivation Rules of *PD* with Identity

(R)
$$\begin{array}{r|l} & \mathbf{P} \\ \triangleright & \mathbf{P} \end{array}$$

(&E)
$$\begin{array}{r|l} & \mathbf{P}\ \&\ \mathbf{Q} \\ \triangleright & \mathbf{P} \end{array} \qquad \begin{array}{r|l} & \mathbf{P}\ \&\ \mathbf{Q} \\ \triangleright & \mathbf{Q} \end{array}$$

(&I)
$$\begin{array}{r|l} & \mathbf{P} \\ & \mathbf{Q} \\ \triangleright & \mathbf{P}\ \&\ \mathbf{Q} \end{array}$$

($\supset$ I)
$$\begin{array}{r|l} & \begin{array}{|l} \mathbf{P} \\ \hline \mathbf{Q} \end{array} \\ \triangleright & \mathbf{P} \supset \mathbf{Q} \end{array}$$

($\supset$ E)
$$\begin{array}{r|l} & \mathbf{P} \supset \mathbf{Q} \\ & \mathbf{P} \\ \triangleright & \mathbf{Q} \end{array}$$

($\sim$ I)
$$\begin{array}{r|l} & \begin{array}{|l} \mathbf{P} \\ \hline \mathbf{Q} \\ \sim \mathbf{Q} \end{array} \\ \triangleright & \sim \mathbf{P} \end{array}$$

($\sim$ E)
$$\begin{array}{r|l} & \begin{array}{|l} \sim \mathbf{P} \\ \hline \mathbf{Q} \\ \sim \mathbf{Q} \end{array} \\ \triangleright & \mathbf{P} \end{array}$$

($\vee$I)
$$\begin{array}{r|l} & \mathbf{P} \\ \triangleright & \mathbf{P} \vee \mathbf{Q} \end{array} \qquad \begin{array}{r|l} & \mathbf{Q} \\ \triangleright & \mathbf{P} \vee \mathbf{Q} \end{array}$$

($\vee$E)
$$\begin{array}{r|l} & \mathbf{P} \vee \mathbf{Q} \\ & \begin{array}{|l} \mathbf{P} \\ \hline \mathbf{R} \end{array} \\ & \begin{array}{|l} \mathbf{Q} \\ \hline \mathbf{R} \end{array} \\ \triangleright & \mathbf{R} \end{array}$$

($\equiv$ I)
$$\begin{array}{r|l} & \begin{array}{|l} \mathbf{P} \\ \hline \mathbf{Q} \end{array} \\ & \begin{array}{|l} \mathbf{Q} \\ \hline \mathbf{P} \end{array} \\ \triangleright & \mathbf{P} \equiv \mathbf{Q} \end{array}$$

($\equiv$ E)
$$\begin{array}{r|l} & \mathbf{P} \equiv \mathbf{Q} \\ & \mathbf{P} \\ \triangleright & \mathbf{P} \end{array} \qquad \begin{array}{r|l} & \mathbf{P} \equiv \mathbf{Q} \\ & \mathbf{Q} \\ \triangleright & \mathbf{P} \end{array}$$

($\forall$I)
$$\begin{array}{r|l} & \mathbf{P}(\mathbf{a}/\mathbf{x}) \\ \triangleright & (\forall \mathbf{x})\mathbf{P} \end{array}$$

provided that
(i) **a** does not occur in an open assumption.
(ii) **a** does not occur in $(\forall \mathbf{x})\mathbf{P}$.

($\forall$E)
$$\begin{array}{r|l} & (\forall \mathbf{x})\mathbf{P} \\ \triangleright & \mathbf{P}(\mathbf{a}/\mathbf{x}) \end{array}$$

($\exists$I)
$$\begin{array}{r|l} & \mathbf{P}(\mathbf{a}/\mathbf{x}) \\ \triangleright & (\exists \mathbf{x})\mathbf{P} \end{array}$$

($\exists$E)
$$\begin{array}{r|l} & (\exists \mathbf{x})\mathbf{P} \\ & \begin{array}{|l} \mathbf{P}(\mathbf{a}/\mathbf{x}) \\ \hline \mathbf{Q} \end{array} \\ \triangleright & \mathbf{Q} \end{array}$$

provided that
(i) **a** does not occur in an open assumption.
(ii) **a** does not occur in $(\exists \mathbf{x})\mathbf{P}$.
(iii) **a** does not occur in **Q**.

Truth-tree Rules for *PL* with Identity

(&D)
$$\mathbf{P} \& \mathbf{Q} \checkmark$$
$$\mathbf{P}$$
$$\mathbf{Q}$$

(∼ &D)
$$\sim (\mathbf{P} \& \mathbf{Q}) \checkmark$$
$$\sim \mathbf{P} \qquad \sim \mathbf{Q}$$

(∨D)
$$\mathbf{P} \vee \mathbf{Q} \checkmark$$
$$\mathbf{P} \qquad \mathbf{Q}$$

(∼ ∨D)
$$\sim (\mathbf{P} \vee \mathbf{Q}) \checkmark$$
$$\sim \mathbf{P}$$
$$\sim \mathbf{Q}$$

(⊃ D)
$$\mathbf{P} \supset \mathbf{Q} \checkmark$$
$$\sim \mathbf{P} \qquad \mathbf{Q}$$

(∼⊃ D)
$$\sim (\mathbf{P} \supset \mathbf{Q}) \checkmark$$
$$\mathbf{P}$$
$$\sim \mathbf{Q}$$

(≡ D)
$$\mathbf{P} \equiv \mathbf{Q} \checkmark$$
$$\mathbf{P} \qquad \sim \mathbf{P}$$
$$\mathbf{Q} \qquad \sim \mathbf{Q}$$

(∼≡ D)
$$\sim (\mathbf{P} \equiv \mathbf{Q}) \checkmark$$
$$\mathbf{P} \qquad \sim \mathbf{P}$$
$$\sim \mathbf{Q} \qquad \mathbf{Q}$$

(∼∼ D)
$$\sim\sim \mathbf{P} \checkmark$$
$$\mathbf{P}$$

(∼ ∃D)
$$\sim (\exists \mathbf{x})\mathbf{P} \checkmark$$
$$(\forall \mathbf{x}) \sim \mathbf{P}$$

(∃D)
$$(\exists \mathbf{x})\mathbf{P} \checkmark$$
$$\mathbf{P}(\mathbf{a}/\mathbf{x})$$
where **a** is a constant foreign to the branch on which $\mathbf{P}(\mathbf{a}/\mathbf{x})$ is entered.

(∃D2)
$$\exists \mathbf{x})\mathbf{P} \checkmark$$
$$\mathbf{P}(\mathbf{a_1}/\mathbf{x}) \qquad \ldots \qquad \mathbf{P}(\mathbf{a_m}/\mathbf{x}) \qquad \mathbf{P}(\mathbf{a_{m+1}}/\mathbf{x})$$
where $\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_m}$ are the constants that already occur on the branch on which ∃D2 is being applied to decompose $(\exists \mathbf{x})\mathbf{P}$ and $\mathbf{a_{m+1}}$ is a constant foreign to that branch

(∀D)
$$(\forall \mathbf{x})\mathbf{P}$$
$$\mathbf{P}(\mathbf{a}/\mathbf{x})$$

(∼ ∀D)
$$\sim (\forall \mathbf{x})\mathbf{P} \checkmark$$
$$(\exists \mathbf{x}) \sim \mathbf{P}$$

(= D)
$$\mathbf{a} = \mathbf{b}$$
$$\mathbf{P}$$
$$\mathbf{P}(\mathbf{a}//\mathbf{b})$$
where **P** is a literal containing **b**.